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Hints respecting the Domestic Cat.

As it is not my intention to write the natural history of the domestic cat. I shall not assert that the wild cat, which is confined to the woods of Europe and Asia, is the parent stock of the whole race ; nor will I attempt to decide on that particular species which was brought by the Indians to Columbus when he discovered America. Thus much, however, I may say, that wild cats are found, with but little variety, in almost every country and climate. My present design confines me to the common house cat, called by the familiar name of tortoise-shell,* tabby, or puss. Though Sonini† has given puss traits of character which place him in a most amiable and interesting point of view, I feel compelled, though I do it with great reluctance, to pronounce him ungrateful, ungenerous, and deceitful ; though with all these abominable qualities, an important domestic. But this is all foreign to the subject : My intention is to inquire into the *cause* of some signs which the cat exhibits, at particular stages of the weather.

Linnæus gives, as one of the specific characteristics of the

* It has been asserted that the tortoise-shell cat is indigenous to Spain, and that it constitutes a distinct variety of the common species ; this opinion, I have good reason to believe, has no sufficient ground for its support.

† See Sonini's Travels in Egypt, with *Bonaparte*.

domestic cat, that he washes his face and behind his ears with his fore feet at the approach of a storm.* However strange or fanciful this may appear, there is, I think, no doubt of the fact. I have observed it often, and have found it an unerring prognostic of falling weather; in winter of snow, and in summer of rain. I shall now offer what I think a sufficient explanation of this remarkable fact. To use the language of chemists, we know that some animals have a greater capacity for electricity, and exhibit much stronger signs of it, than others. The gymnotus electricus, or electrical eel, will give a *shock* to any number of persons in the same manner as the leyden jar. The electrical properties of the torpedo, or cramp-fish, are so remarkable, that for a long time they were considered fabulous. Some other fish might be mentioned, as possessing this property. These examples are produced to show that electricity may exist in the bodies of animals, in large quantities, without disturbing their ordinary functions. The experiment of producing sparks of electrical fire by rubbing the back of a cat is familiar to almost every one ;† the glittering of the eyes by night may, I think, be also stated as another proof of the presence of the fluid. The ears of the cat, and some portions of the face, are without hair; such places are good conductors of electricity; but the whole body of the animal being enveloped in a hairy covering, is a non-conductor. If, therefore, the fluid escapes, or passes off in any considerable quantity, it must be at the ears or face. This can be proved by experiment: Upon rubbing the back of the animal in favourable weather, and presenting your knuckles to the ear, a spark will be

* See Turton's, Linnæus, vol. i. p. 49.

† It is said that black cats give out more electric fire than those of any other colour. If this be a fact, may it not lead to some important results in the construction of electric machines. It has been found that the power of a white glass cylinder, is considerably increased by coating the inner surface with a coloured electric, such as rosin, or bees-wax.

received ;—a spark, however, cannot be obtained from any part of the body, a few scintillations only following the hand in the act of rubbing. Cats, we know, have a natural antipathy to water and moist air, and delight in dry and warm situations.* It is hardly necessary to state, that a damp atmosphere is one of the best conductors of electricity, or that a dry one is necessary for its collection and retention. From what has now been advanced, the conclusion is obvious. The vapour or humidity of the air, serving as a conductor, draws off or absorbs the electric matter from the animal, and the fluid passing off with more readiness, and in the greatest quantity, at the ears, must occasion sensations in that particular part, either agreeable or troublesome, which induces the motion of the fore foot as above stated. That some animals are more readily affected by changes in the atmosphere than others, and that they exhibit signs of these variations cannot be doubted. The difficulty is, since we are wholly unacquainted with the cause of muscular motion, to explain with clearness and precision how this takes place. I will conclude by observing, that Mr. J. Taylor, in an interesting little volume, called the *complete weather guide*, has given a chapter on the common and familiar signs exhibited by animals which indicate approaching changes of weather; in this chapter he passes, without notice, our friend the CAT.†

J. G.

Princeton, June 4, 1818.

* I cannot here help remarking, that the cat is particularly fond of fish. On giving this propensity, nature seems to deviate, from her usual course; for, considering the antipathy of the cat to water, it can rarely indulge its natural appetite.

† See Taylor's Guide, second London edition, p. 22.

To the Editor of the Scientific Journal.

Sir,

I have some observations to make on Y's solution of question 17. The second case of it is right, but the first and third cases, I conceive, have nothing to do with the question: for, in these cases, the three angles of the triangle are upon only three sides of the square, and, therefore, the magnitude and situation of the triangle are wholly independent of the fourth side, so that, instead of these cases answering the question proposed, they answer the following, viz. having two straight lines which are parallel to each other, and perpendicular to a third, knowing the part of the third line intercepted between them, it is required to describe an equilateral triangle, which shall have its angular points upon the three straight lines. I beg to refer Y. to the first definition of book 4, Playfair's Euclid; when he has read that, I think he will excuse me for making the above remarks on his solution.

In his answer to the 12th philosophical question, he seems to have misapprehended the nature of it: his reasoning is certainly ingenious, but he does not touch upon that part of it in which the difficulty lies. If he will examine the corollaries again, he will find that Newton supposes the periodical times about the different centres of force to be equal. The question then is, how does this appear to be right? I wish to have the question proposed again, in the manner in which I have now stated it. I trust that Y. will excuse me for making the above remarks on his solutions; I do assure him that I am very friendly to him, but I presume he will agree with me, that "truth ought not to be sacrificed to friendship."

Yours with respect,

ZERO.

Qu. 34, *answered by Zero.*

Conceive the cylinder placed as in the question, it is then required to find the solidity, and surface, of that part of the perforating cylinder included by the other. Imagine a sphere of the same diameter as the cylinders to be inscribed within the solid under consideration, and it will evidently be touched by each of the cylinders. It is further manifest, since the diameters of the globe and cylinders are equal, and the axes of the cylinders are perpendicular to each other, that the cylinders will touch the globe in two great circles of it, which are at right angles to each other. It is further obvious, that those sides of the two cylinders which meet each other will be at right angles to one another. Conceive, then, any plane to be drawn parallel to the plane of the axes of the cylinders, cutting the solid and its inscribed sphere, then will it cut the surface of each cylinder in two straight lines, which will be parallel to each other, and which will touch the corresponding section of the sphere, which is a circle. It is manifest, therefore, since the plane cuts the superficies of one cylinder in two straight lines, which are perpendicular to the two straight lines in which it cuts the superficies of the other, and since these four straight lines touch the circle cut out of the sphere by the aforesaid plane, that the section of the solid by the above-mentioned plane will be a square described about the corresponding circle of the sphere ; therefore, the section of the solid is to the corresponding section of the sphere, as the area of a square to its inscribed circle ; hence, since this is true of any section which is formed as above described, it follows, by the method of limits, that the solid is to the inscribed sphere, as the area of a square to that of its inscribed circle. It is manifest, from what has been said above, that the common section of the above-mentioned plane, and the superficies of the solid,

form the four sides of a square, which touch the corresponding circle of the sphere ; and that this is true of all sections made as above-described : hence, the section of the surface of the solid, is to the corresponding section of the surface of the sphere, as the four sides of a square to the circumference of its inscribed circle : hence, by the method of limits, the four sides of a square are to the circumferences of its inscribed sphere, as the surface of the solid to that of its inscribed sphere.

Some account of the Leech.

Mr. Rafinesque, in the 2d No. of your Journal, has given a description of a new genus of North American Aquatic Worms, which he calls Pettostoma. The species which he mentions, as a type of the genus, he calls Testudaria, which was found in the Delaware attached to a water tortoise—its vulgar name, he says, is the *turtle leech*. I have great reason to believe that Mr. R. has hastily formed a new genus for this worm ; however, as I have not seen the animal he describes, I cannot possibly determine the question. The following are the reasons why I do not think the *turtle leech* entitled to form a new genus. Early as April last, I discovered on three different individuals of the common water tortoise (testudo punctata) a kind of leech, which, on first examination, I had no doubt was the pettostoma testudaria of your correspondent. The narrow articulations of the back, the variegated flexious lines, the shield mouth, and the row of brownish spots along the sides—all very nearly agreed with his description ; besides, in trying to remove the worm from the tortoise, I found that it adhered only at one end. Had I taken no farther notice of the animal, I should have concluded that it ought not to be placed in the genus *hirudo*, the essential character of which is, that it is

truncated, or sticking at both ends, but upon placing the worm in a jar of water, where I kept it for nearly three months, in company with some others of the same family, and some near relations, of whom I shall speak presently, I found that both ends were sticking, though it made but little use of one, except in its progressive motion along the sides of the jar. To whatever genus Mr. R.'s animal may belong, I am satisfied that mine is an *hirudo*. As it is not described in any of the books, as far as I know, I would call it hirudo testudaria, thanking your correspondent for his specific name. I have seen four of these animals, and in every instance there has been some little difference in the arrangement and shades of their colours; the difference, however, is not sufficient to constitute a distinct species. In its manners, it does not resemble the other individuals of the leech tribe which I have seen: it is more sluggish, very rarely moving about; it never swims in the manner of an eel, as most of the species do, and I could not perceive that it was more restless before a change of weather than at other times, a peculiarity which is said to characterise the leech tribe. I have another kind of leech, which is different from any I have seen described. It is very frequent in shallow waters in the early part of Spring. The body, which, like most other species, is composed of numerous annular wrinkles, or folds, is round, and, when extended, is slightly tapering from the tail towards the head, or mouth. The eyes are not visible, and the body is truncated at both ends. Its progressive motion is effected by firmly fixing one end to any substance, in the manner of a sucker, and drawing the other end up to it, the body assuming the form of an arch. This end is now fastened, and the other thrown out, and so on, the head always preceding; it often, however, swims through the water in the manner of an eel. The colour of the whole body is uniform, being olive, which is sprinkled, here and there, with a few spots of black. I know that the colour in almost all the species of hirudo is va-

riable, owing to age and to the seasons, and, possibly, this may only be found to be a variety of the hirudo sanguisugo, or horse leech. The length is between five or six inches when extended. The animal rests in the glass vessel in which it is kept, by fastening the tail to the side, the head being drawn close to it, but not attached to the glass. They are fond of light, and are, I suppose, a diurnal animal. When a candle is placed near the vessel, they gather to that side which is nearest it. Their food is animal juices; they suck pieces of meat, thrown into them, whether raw or cooked. I suspect it will suck blood with too great avidity, and in too large quantities, to be applied safely to medicinal purposes. If a new species, it might be called hirudo multevora, in compliment to its great appetite.

My principal object in this communication is, to notice the last-mentioned leech as a weather guide. From the observations I have made, I have no doubt that they exhibit particular changes before the variations of weather: but that frost and snow, rain and wind, are all designated by some peculiar sign given by the worm—*from my experience*, I have no reason to believe. The two following conclusions are all that I could draw from its motions: First—That when the weather was clear, the animal lay motionless at the bottom of the jar. Second—That a little before, and during the continuance of wind, the animal was restless.

Mr. W. Peck, in a letter to the editors of the Philosophical Magazine, November 5, 1809, states, that the animals in his possession exhibited the following appearances:

First—When the leech lies motionless at the bottom of the glass, and is frequently in a spiral form, the weather in summer will be serene and beautiful; the same denotes clear frosty weather in winter.

Second—If it creeps up to the top of its lodging, it will rain within twenty-four hours in summer, and snow in winter.

Third. When the leech gallops through its limpid habitation with swiftness, it denotes wind, and seldom rests until it blows hard.

Fourth. When the leech lodges almost constantly out of the water, and discovers uncommon uneasiness, in violent throes, and convulsive-like motions, a storm of thunder and rain will succeed.

Mr. James Stockton, of Malton, made some observations on the horse-leech, as a *weather-glass*, an account of which is published in the Port Folio, for June, 1818; but as these appear to be copied from Mr. Peck's, I shall not insert them.

J. G.

Princeton, July 16, 1818

Qu. 13. *Answered by Zero.*

Draw from the centre of the earth a straight line to the eye of the spectator; then, since the latitude of the place is known, the part of this line intercepted between the centre of the earth and its surface can be found. Let, also, a plane be drawn to touch the earth at the point where this line meets its surface, which can readily be done, for it is to be drawn perpendicular to the direction of falling bodies at that point. Find a third proportional to the distance of the eye from the earth's centre, and the distance of the point of contact of the tangent plane from the centre; cut off then on the line joining the eye and the centre, a distance towards the eye, equal to this third proportional; then, through the point of section, draw a plane parallel to the tangent plane, and the line in which this plane cuts the surface of the spheroid, will be the curve required. For, conceive any plane to be drawn through the eye and the centre of the earth, and its section with the earth's surface,

will manifestly be an ellipse, having its centre at the centre of the earth. It will also cut the tangent plane in a straight line, which will touch the ellipse at the vertex of the semi-diameter drawn to the point of contact of the tangent plane, and it will cut the plane which was drawn parallel to the tangent plane in a straight line, which will be parallel to the tangent. Therefore, this line is ordinately applied to that diameter of the ellipse which passes through the eye. Conceive two tangents to be drawn from the eye to the ellipse, then join their points of contact, and the joining line will be parallel to the tangent at the vertex of the semi-diameter which passes through the eye, and this semi-diameter will be a mean proportional between the distance of the line joining the points of contact from the centre, and the distance of the eye from the same; but, (by construction,) this semi-diameter is also a mean proportional between the distance of the eye from the centre, and the distance of the common section of the plane drawn parallel to the tangent plane and the plane which cuts the ellipse. Therefore, the distance of the line of common section from the centre, is equal to the distance of the line joining the points of contact of the tangents from the centre, and both these lines, as has been proved, are parallel to the tangent at the vertex of the diameter which passes through the eye. Therefore, these lines coincide, and the ellipse we have been considering is any one whose plane passes through the eye, and the earth's centre; whence, the locus of the points of contact of all the straight lines which can be drawn from the eye to touch the earth, and consequently of all the tangent planes, lies in the common section of the plane which we drew above, parallel to the tangent plane. But this plane cuts out an ellipse in the case specified in the question, whose distance from the centre of the earth and position are known.

Another answer to question 13, *by C. West-Point.*

The earth being an ellipsoid, the curve of contact of a cone with it is an ellipse; this we may conclude from this general principle, that if a cone be tangent to a surface of a certain degree, the curve of contact is contained in another surface of an immediately inferior degree. But it may be demonstrated geometrically as follows: Upon the great circle of the ellipsoid, let us conceive a sphere to be described, it is obvious that every ordinate of the ellipsoid bears a constant ratio with the corresponding one of the sphere; for every section made by a vertical plane in the two surfaces will be an ellipse and a circle, which have that known property, consequently all the vertical sections of the ellipsoid are similar. Let us now conceive, through the given point, a cone, tangent to the ellipsoid, and through a point raised above it in the ratio of the two ordinates of the surfaces, another cone tangent to the sphere: then will any plane, drawn vertically through the common ordinate of these two vertices, cut the two cones in two elements, tangent to the sections made in the respective surfaces; and the two points of contact are in the same ordinate, (vide Conic Sections), consequently the two curves of contact will be vertically above each other in a given ratio. But the curve of contact of a cone and sphere is a circle, a plane curve; therefore the curve of the ellipsoid must be a plane curve also; for if the ordinates of a plane be increased proportionally, they will form another plane; consequently, this curve is an ellipse. The planes of the two curves always intersect in the plane of the equator.

Qu. 14. *Answered by Zero.*

This question depends for its solution on the foregoing, in which it has been shown, that the curve of contact is an ellipse,

in every situation, except when the eye is at one of the poles, where it is a circle. That the dip is not constant, when the height of the eye is the same, may be thus shown. Conceive the spectator to be at one of the poles, then will the curve of the tangent planes drawn from the eye be a circle, by what has been shown in the last solution; therefore, the angle formed by the tangent planes in that situation, and the perpendicular to the earth's surface, or polar axis, will be constant; hence, the complement of those angles or the dip of the horizon will be the same. Let, now, the spectator be at the equator; then will the curve of contact be an ellipse, having its axes in the same proportion as the axes of the earth; therefore, the angles formed by the tangent planes, and the perpendicular to the earth's surface, which will be the equatorial diameter produced, will be unequal; consequently, the height of the eye remaining the same at the pole and the equator, the dip will not always be the same; for it is constant for all the tangent planes at the pole, but variable for the tangent planes at the equator. For the complements of the angles made by the tangent planes at the equator, and the equatorial diameter drawn to the eye, which is the dip, is variable. By a like process, it may be shown that the dip is not constant at points between the poles and the equator.

Solution to Qu. A. by N. King, Esq.

Considered absolutely, the shortest distance between the two points is the *straight line*—this lies in the sphere. The shortest, on the surface, must be an arc of a circle, the *nearest* to the straight line that can be drawn. A connecting line, not straight, cannot be lengthened without departing still further from straightness, nor shortened but by approach and assimilation to the straight line. In shortening, it would run directly into, and be identified with, that line, if not impeded. By the same

principle, when limited to the surface, it places itself in the plane of a great circle, because in that position it is every way *nearest* to the chord, and most like it, in other words, because it is then shortest.

We may take the two points in a meridian, in equal N. and S. latitudes, and call the straight line between them, A. We may note the point, on the plane of the equator, where A is bisected. Let a circle in that plane, and touching the equator, be described on that point as a centre. *The contact will be in the meridian of the two points.* The touching radius, and every other radius produced to the equator, will be a leg of a right angled triangle—of which, on each side of the plane aforesaid, the constant leg will be half A—and the hypothenuse, the chord of an arc extending from one of the two points to the equator. Call the triangle that has the radius, B—and one of the other triangles, C. The variable leg in C is longer than the radius in B. Then the hypothenuse of C is longer than the hypothenuse of B; and the longer hypothenuse will be the chord of a *longer arc.* C's arcs, North and South of the equator, may be parts of different great circles. Those of B are in one—a route shortest, because, taken entire, it proves to be *nearer* than any other to the straight line.

On Heat.

Heat is one of the effects of fire or caloric, indicated by an increase of temperature, and the sensation it produces on the organs of feeling. Various hypotheses have been advanced, both by ancient and modern philosophers, to account for the phenomena of heat, some considering it as a body, *sui generis*, and others only as a *quality* of bodies. We shall not attempt to detail the different notions that have been advanced on this sub-

ject ; we shall only state a few of the opinions which have been entertained by the best modern philosophers, where it may be observed, that the question respecting the materiality of light is not fully and satisfactorily determined. Macquer, in the article fire, in his Chemical Dictionary, regards heat as a fluid, and the only essentially fluid body in nature, and consequently, as the cause of fluidity in other bodies ; and although he speaks of the great facility with which it penetrates bodies, or separates from them, and infers from these facts the infinite smallness of the integrant parts of fire, meaning by this term the cause of heat, or what is now called caloric ; yet, in the appendix to his dictionary, when treating of the same subject, he expresses an opinion directly opposite to that before given, and calls heat only "a mode of existence of a material substance." In this elaborate dissertation, he institutes an *analogy*, which he sustains with considerable ingenuity, between the communication of heat and the communication of motion, and deduces from the known and acknowledged laws of the latter, the similarity of the former, and consequently maintains that heat is really nothing else but the motions of the parts of heated bodies. "I have hitherto," says he, "been of opinion with most philosophers, that heat is a particular kind of matter, so subtile as to penetrate all bodies, and to separate their parts when put in motion by light or percussion, and that this being was the true matter of fire ; but the above reflections have suggested a different opinion. There certainly is a matter of fire, and it is pure light, which is a material substance, whose existence cannot be questioned. But we cannot say the same of heat : the causes which excite it, and the effects which it produces, do not prove, or even suppose, the existence of a peculiar matter. They all concur, on the contrary, to indicate that it is only an accident, a modification of which some bodies are susceptible, and consists merely in the intestine motion of their constituent parts, which may be produced, not only by the im-

pulse of light, but also by all frictions and percussions of any bodies."

Count Rumford, from two considerations, infers the immateriality of heat, first, from his frequent failures in attempting to discover if it had weight; second, from the possibility which he conceives he has demonstrated, of obtaining an inexhaustible supply of heat from a given quantity of insulated matter. It is well known, that on the freezing of water, a portion of heat is given out, sufficient to raise the temperature of an equal quantity of water no less than 140 degrees; that is, from 32 to 172 degrees. If, therefore, heat were a ponderable substance, it is reasonable to imagine that water would become lighter when frozen in a vessel hermetically sealed. The Count, accordingly, made a number of experiments to ascertain this point; but he never could discern the least difference between the weight of the water before freezing, and that of the ice when frozen. From this he makes his first deduction of the imponderability of heat, and draws the more extensive, but less cautious inference, that all future attempts to discover any effect of heat upon the apparent weights of bodies will be fruitless. *Philosophical Transactions*, 1799. *Part* 2.

The other deduction, above-mentioned, was made from the following experiments. The Count procured a cylinder of brass, bored like a cannon, by means of the engine used for that purpose in the arsenal at Munich; a blunt borer, or flat piece of hardened steel, was kept with one of its extremities strongly pressed against the bottom of this hollow cylinder, while the latter was turned swiftly about its axis. In one experiment, the cylinder was covered on the outside with a coating of thick flannel; in another, the borer was made to work through a collar of leather; in a third, the cylinder was wholly immersed in water, the borer still working through the leather collar; in a fourth, the leather was removed, and the water had access to the interior of the cylinder, where the fric-

tion took place. In all these cases, a very considerable quantity of heat was produced, much more than he conceived it possible for the machinery to supply; and at one experiment, heat was generated by the friction, sufficient to cause about twenty-six pounds.

(To be continued.)

NEW PHILOSOPHICAL QUESTIONS.

Qu. F. *By Zero.*

Granting the truth of the laws laid down by writers on hydrostatics, it is req. to determine the nature of the vessel, whose axis being perp. to the horizon, and having an indefinitely small hole in its vertex, may be such, that, if filled with water, or any other liquor, and suffered to discharge itself, the distances through which the surface of the fluid descends down the axis, may be equal in equal times?——*See Marrat's Mechanics, under Clepsydræ.*

Qu. G. *by N. King, Esq. Hamilton.*

Two balls, each one inch in diam. are rolling in the same right line, on a horizontal plane, to which their axes are parallel; the dist. of their centres is 10 inches, (which let be the radius:) now, if the foremost be instantly slipped one minute from its course, and continues to roll preserving its distance and axis, the plane of its equator still bisecting the other ball; required, the length of the curve line described by the centre of the ball disturbed?

Qu. H. *by Mr. O'Shaunessy, Albany.*

On the Mechanics and Farmers Bank of this city, stands a parabolical cupola, the diameter of its base being 21, and alt. 12 feet; from the centre of this base a cylindric rod, 11 feet long, is placed against the concave surface: now, supposing the rod to descend along the curve with a uniformly accelerated velocity of one foot the first second, and so on, according to the squares of the times; required, the time of its descent?

MATHEMATICAL CORRESPONDENCE.

MATHEMATICAL QUESTIONS ANSWERED.

Qu. 23. *Answered by B. Salem.*

Upon a cursory examination of the 6th number of your very useful Journal, page 117, I find the equ. $d\left(\frac{ydx}{dy}\right)=\surd(dx^2+dy^2)$, was integrated by series, and the opinion advanced that the relation between the ordinate and curve, could not be expressed in finite algebraic terms; but as it may be expressed in finite terms by means of hyp. logs. I have concluded to send a sketch of a solution. Put $\frac{dx}{dy}=p$; then, $d(yp)=dy\surd(1+p^2)$, from which, by developement, we easily get $\frac{dy}{y}=\frac{dp}{\surd(1+p^2)-p}$, or $\frac{dy}{y}=dp\left(p+\surd(1+p^2)\right)$, whose integral taken so that $y=a$ when put $p=o$, becomes $2\log.\frac{y}{a}=p^2+p\surd(1+p^2)$ $+\log.(p+\surd(1+p^2)$, as is easily proved by taking the differential. Then, for any given value of p, (which represents the tan. of the angle formed by the curve and ordinate,) we may obtain y, and then $z=py$. The value $p=\frac{z}{y}$, being sub-

stituted in the 1. equ. gives $2 \log. \frac{y}{a} = \frac{z^2}{y^2} + \frac{z}{y}\sqrt{(1+\frac{z^2}{y^2})} +$ $\log. \left(\frac{z}{y} + \sqrt{(1+\frac{z^2}{y^2})}\right)$, which is the integral of the equ. treated of, giving the relation between z and y. This expression may, by connecting the logs. and reducing the surds, be put under the following form, $\log. \frac{y}{a^2}\left\{\sqrt{(z^2+y^2)} - z\right\} = \frac{z}{y^2}$ $\left\{\sqrt{(y^2+z^2)} + z\right\}$, which also expresses the relation between the ordinate and curve. If, for brevity, we put $p + \sqrt{(1+p^2)} = w$, or $p = \frac{w^2-1}{2w}$, the first integral will become $2 \log. \frac{y}{a} = pw + \log. w$, and if $\log. e = 1$, this becomes $\frac{y^2}{a^2} = we^{pw}$ $= we^{\frac{1}{2}(w^2-1)}$; therefore $y = aw\, e^{\frac{1}{2}\frac{1}{2}(w^2-1)} e^{\frac{1}{2}(w^2-1)}$. Mult. by p gives py, or z, therefore $z = \frac{1}{2}\, a.\, w^{-\frac{1}{2}(w^2-1)}.\, e^{\frac{1}{2}(w^2-1)}$, Thus we have y and z expressed in terms of w, and we may observe that if $p = \cot. v$, $w = \cot. v + \operatorname{cosec}. v = \frac{\cos. v + 1}{\sin. v}$ $= \frac{2 \cos. \frac{1}{2} v^2}{2 \sin. \frac{1}{2} v \cos. \frac{1}{2} v} = \tan. \frac{1}{2} v$, so that $w = \tan. \frac{1}{2}$ the angle included by the tan. and absciss, and we may express y and z in finite terms of this tangent. Lastly, since $x = \int p dy = \frac{a}{v} \int \left(\frac{w^2-1}{w}\right)$. $d\left\{w^{\frac{1}{2}\frac{1}{2}(w^2-1)}.\, e^{\frac{1}{2}\frac{1}{2}(w^2-1)}\right\} = \frac{a}{4} \int (w^4-1)\, w^{-\frac{3}{2}\frac{1}{2}(w^2-1)} e^{\frac{1}{2}(w^2-1)} dw,$

if we develope $e^{\frac{1}{2}(w^2-1)}$ according to the powers of w^2-1 (which is o at the commencement of the curve, where $w=1$) we may obtain a series for x in terms of w, so that x, y and z will be obtained by means of u.

Another answer to the same, by Analyticus, New-York.

Let x, y, z, be the abscissa, ordinate, and curve, and $t = \frac{dx}{dy}$; then, by the qu. $\frac{ydx}{dy} = z$, that is $ty = z$; whence $t\,dy + y\,dt = dz = dy\sqrt{(1+t^2)}$, therefore, $\frac{dy}{y} = \frac{dt}{\sqrt{(1+t^2)} - t} = t\,dt + dt\sqrt{(1+t^2)}$; the integral of which is $h.l.\frac{y}{c} = \frac{1}{2}t^2 + \frac{1}{2}t\sqrt{(1+t^2)} + \frac{1}{2}h.l.(t+\sqrt{(1+t^2)})$. To find x, put $t+\sqrt{(1+t^2)} = 2u^2$, whence $tu^2 = u^4 - \frac{1}{4}$, and $t = u^2 - \frac{1}{4}u^{-2}$; and, by substitution, the preceding equ. in y and t, becomes $h.l.\frac{y}{c} = tu^2 + h.l.\,u\sqrt{2}$, or $h.l.\frac{y}{cu\sqrt{2}} = tu^2$, and, consequently, $y = \sqrt{2} \times cu \times e^{tu^2} = {}^2\sqrt{2} + c\,u.\,e^{u^4 - \frac{1}{4}}$, that is $y = \sqrt{2} + c\,e^{-\frac{1}{4}}\,u\,e^{u^4}$; but $dx = t\,dy$, therefore $x = c' + ty - fydt$ which last integral, by substituting for y and t their values in u, gives us $x = c' + ty - 2\sqrt{2}.\,c\,e^{-\frac{1}{4}} \int u^2\,du.\,e^{u^4} - \frac{1}{2}\sqrt{c\,e^{-\frac{1}{4}}} \int u^{-2}\,du.\,e^{u^4}$. Thus the prob. is reduced to the integration of the formulæ $u^2\,du.\,e^{u^4}$ and $u^{-2}\,du\,e^{u^4}$; this integration, however, has not yet been effected without infinite series.

Another by C..West-Point.

I shall treat this problem in a general way, by finding the curve which bears a constant ratio to its sub-tangent. Let that ratio be m, then we have $\frac{y\,dx}{dy} = mz \ldots (1)$. Make dy constant, then by differentiation we find $dy\ dx + y\,d^2x = m\,dy\,dz$, or $\frac{dy}{y}.\ dx + d^2x = m\,dz.\ \frac{dy}{y}$; but $\frac{dy}{y}$, by equ. $(1) = \frac{dx}{mz}$, whence $\frac{dx^3}{mz} + d^2x = \frac{mdz\,dx}{mz}$, or $dx^2 + md\,d^2x = mdzdx$. Put $\frac{dx}{dz} = p$, then $\frac{d^2x}{dz} = dp$, and the last equ. becomes $mzdp + p^2\,dz = mp\,dz$, or $\frac{dp}{mp - p^2} = \frac{dz}{mz}$, whose integral is $\frac{1}{m}\,h.\,l.\,\frac{p}{m-p} + \frac{1}{m}\,h.\,l.\,C = \frac{1}{m}\,h.\,l.\,z$, or $\frac{pc}{m-p} = z \ldots (2.)$ To find C, we may observe that $p = \frac{dx}{dz}$ is the cos. of the angle which the tan. to the curve makes with the axis, and at the beginning of the curve where $z = o$, a small increment may be taken for its tan. and must bear to the corresponding increment of the abscissa, which is its sub-tangent, the given ratio m; then $\frac{dx}{dz} = m = p$, whence $C = o$, conseq. $z\,(m-p) = o$, therefore, $p = m$, or $\frac{dx}{dx} = m$, and $dx = mdz = m\surd(dx^2 + dy^2,)$ or squaring $dx^2 = m^2\,dx^2 + m^2\,dy^2$, whence $dy = \frac{\surd(1 - m^2)}{m}\,dx$, whose inte-

gral is $y = \frac{\sqrt{(1-m^2)}}{m} x + B$, where B is unnecessary. This equ. is that of a line whose angle with the axis has for its tan. $\sqrt{(1-m^2)} \div m$, and for its cos. the given angle m. It is obvious that this line has the specified property, and that when $m = 1$, the equ. becomes $y = B$, which is the equ. of a parallel to the axe, and does not intersect it as Y has remarked. As my solution is general, I do not believe that any other curve has that property, but in operating otherwise I found equ. which must be identical with my result, but which involve so many complicated terms, that it would be difficult to compare them together, but, in fact, its unnecessary.

ZERO—Finds $e^{\frac{z}{\sqrt{(z^2+y^2)}}} \times \left\{ \sqrt{(z^2+y^2)} + z \right\} = c^z$ y^3 which is a finite equ. between y and z; he then determines $c = y^{-1}$ when $z = o$, and observes, I should have sent this result when I sent the answer to the qu. but I concluded that the proposer's intention was to find an equ. between the absciss and ordinate.

Qu. 28. *Answered by Mr. R. G. Hazard, Bristol,*

Put the diameter of the sphere $= a$, x the alt. of the seg. and $p = 3.1416$ then $(3ax - 2^2) \times \frac{1}{6} px =$ the solidity, and $pax =$ curve surface of the seg. therefore, their ratio $(ax - 2x^2) \div 6a$ must be a max. in fluxions, and reduced $x = \frac{3}{4} a$.

Qu. 29. *Answered by Y, New-Haven.*

This qu. will be best solved by finding in how many ways a given number can be composed out of the points on the surfaces of n dice; for, when this is done, we have only to divide

it by $6n$, the whole number of compositions, to obtain the chance that the given number will be turned up at a single throw. In the first place, then, if we have but one die, the ways in which the successive number can be turned up will be 1. 1. 1. &c. If we have two dice, let the number on the first be denoted by 1, 2, 3, &c. and those on the second by 1', 2' 3', &c. Two can be formed only by joining 1 to 1': Three—by joining 1 to 2', or 2 to 1': Four—by joining 1 to 3', 3 to 1', or 2 to 2'. Beginning at the other extremity, the same reasoning applies to the successive num. 12. 11. 10. &c. Therefore, the ways in which the num. from 2 to 12, inclusive, may be thrown with two dice, are denoted by the natural series of num. *ascending* to 6, and then *decending* to 1. If we have three dice, the number *three* can be found only by joining the three *ones* ; *four* by joining each of those which produced a 3, in the two first to 1 in the 3d. and that which produced a 2, to 2 in the third ; hence, the whole number is $1 + 2$. Five may be composed by joining those which produced a 2 to 3, those which produced a 3 to 2, and those which produced a 4 to 1 ; hence the number is $1 + 2 + 3$. In the same manner, *six* may be produced in $1 + 2 + 3 + 4$ ways : and in general, the series for 3 dice may be formed from that for 2, by adding the 1 and 2. the 1, 2, 3, &c. in it to the 6th. when the 7 must be added and the first subtracted, the 8 added, and the two first subtracted, &c. to the last. The number for 4 dice may be formed from the last series in the same manner as the last from the preceding, and in this manner, whatever be the value of n, the ways in which the given number can be thrown may be found. For three dice they stand thus,

For the numbers	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
The ways are	1	3	6	10	15	21	25	27	27	25	21	15	10	6	3	1

In this, as in all the other cases, the series decreases alike on both sides from the middle, and ends with one. The six first and six last terms of any series are figurate numbers

of the order denoted by the number of dice. Hence, for these a general solution of the problem may be obtained. Let m, (not greater than 6,) denote the excess of the given number above, $n-1$, or what it wants of $6n+1$. The chance that this number will be turned up in thowing n dice is denoted by $\frac{m}{1}\cdot\frac{m+1}{2}\cdot\frac{m+2}{3}\cdot\cdot\cdot\frac{m+(n-2)}{n}\cdot\frac{1}{1}\cdot\frac{1}{6^n}$.

But if the given number exceed the least, or fall short of the greatest which can be thrown by more than 5, it will be necessary to resort to the method of successive additions, already pointed out.

Qu. 45. *By Mr. J. Laidlaw, Brooklyn*

From these equations here subjoined,
A lady's fortune I must find,
Or else the maid I cannot have;
So gents your friendly aid I crave,
Not doubting but you'll make appear,
How I may gain my lovely dear.

$x^{\frac{3}{2}}+y^{\frac{3}{4}}=265,8008$; $y^{\frac{3}{2}}+z^{\frac{3}{4}}=173,7741$ and $z^{\frac{3}{2}}+x^{\frac{3}{4}}=105,3481$. Where x is the number of eagles, y the pounds, in New-York currency, and z the dollars that constitute the lady's fortune.

Qu. 46. *By C. H.*

Find a number consisting of 3 places, whose digits are in arithmetical progression, which number, if divided by the sum of its digits, the quotient will be 48, and if 198 be subtracted from it, the digits will be inverted.

Qu. 47. *By Omicron.*

A girl exchanges cheeses for hens, at the rate of 2 cheeses for 3 hens; the hens lay each $\frac{1}{2}$ as many eggs as there are cheeses, the girl sells 9 eggs for as many pence as each hen laid eggs, and received for all her eggs, 72 pence; how many hens and eggs were there in all?

Qu. 48. *By C. West Point.*

Prove, that if from any point in the greater axis of a conic section, a tangent be drawn, and an ordinate let fall, from the point of contact; lines drawn from the foot of this ordinate to the points, where a second drawn from the first point intersects the curve, will make equal angles with the axis, or the ordinate.